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ABSTRACT

of the dissertation for the degree Doctor of Philosophy

DISCRETE ADDITIVE, MULTIPLICATIVE AND POVERATIVE DERIVATIVE CONCEPTS AND APPLICATIONS TO SOLUTION OF THE CAUCHY AND BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL DERIVATIVE EQUATIONS

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GENERAL CHARACTERISTIC OF WORK

Actuality of the research theme and degree of processing. It is known that there is a small number of natural phenomena which brings the mathematical model to the problems for discrete differential equations, from them can be shown the finding of the general limit of a arithmetic progression, finding the general limit of a geometric progression, and finding the general limit of a Fibonacci sequence.

The well-known theory for ordinary differential equations with differences for equations was transferred by A.O.Gelfond. In 2010, out of 50 collections published for comprehensive school, the 40^{th} number is devoted to the degree of development of a set of numbers, integrals were determined, each containing two direct and two inverse operations. Finally, discrete integrals were defined here, given by a direct operation, and discrete derivatives, given by an inverse operation.

The discrete additive derivative, since it is given by only the difference, has been very well researched for those kinds of problems.

Despite the fact, 50 years ago the multiplicative derivative and the integral were given in [1] in the case of continuality, unfortunately, recently, research has begun again on problems for such types of equations.

Basically, we have started to research of the problems for equations with a discrete multiplicative derivative.

Finally, in order to give the poverative derivative and integrals in the case of continuality and in the case that new inverse and direct operations are needed to give a discrete poverative derivative and a discrete poverative integral, there is no need for new operations, we ourselves had to implement these operations.

By presenting the discrete multiplicative derivative and the discrete poverative derivative, we have achieved solutions to problems with very complex nonlinear equations, and obtained analytical expressions for these solutions by means of discrete multiplicative and discrete poverative integrals. That indicates about the actuality of the research theme.

The degree of processing of the research theme is at the level of modern mathematics. Thus, using theses of discrete derivatives in all considered issues the obtained nonlinear algebraic equations are solved, and in all cases analytical expressions are obtained for the solution.

Research object. The object of the dissertation is the construction of mathematical models of discrete events and the research of their solutions. As mentioned above, discrete events are arithmetic progression, geometric progression, and Fibonacci sequences.

Research subject. As for the subject of the research, we can say that when it is difficult to investigate the solution both, in problems considered for ordinary differential equations, also in problems for special derivative equations, these problems are discretized with a certain step, the obtained system of algebraic equations is solved. By approaching the step in the solving of a discrete problem to zero, it is possible to come to a certain conclusion about the solution of a continuous problem. Thus, discrete problems also help to solve continuous problems.

The goal and tasks of research. The aim of the work consists of the obtaining of analytical expressions for solving problems for nonlinear complex equations. These expressions are given via discrete additive integral sets, discrete multiplicative integrals (totals) and finally discrete poverative integrals (coefficients). Thus, the solution of all issues in this work is determined analytically. The tasks of the research is consist of conveying the results obtained by the researcher to the world of mathematics. The main task of the research is concrete and to make the obtained result meaningful.

Research methods. In the dissertation work, using the definitions of only discrete additive derivative and discrete additive integral, discrete multiplicative derivative and discrete multiplicative integral, discrete poverative derivative and discrete poverative integrals have been used the methods of linear algebra, analytical geometry and mathematical analysis.

The main provisions for the defense:

1. In the case of continuity, if two consecutive inverse operations are given, the derivatives are giving by one inverse operation.
2. In the case of continuity, if the integral is given with the help of two consecutive straight operations, discrete integrals are giving through one straight operation.
3. Solution of Cauchy and boundary problems for a second-order equation with two different derivatives.
4. Solution of Cauchy and boundary value problems for the third-order equation of three different derivatives.

Scientific newness. Compact analytical expressions for solutions of boundary value and Cauchy problems for very complex nonlinear equations of discrete analysis were obtained. For various differential equations containing discrete derivatives up to the third order, problems have been considered in nonlinear conditions.

Theoretical and practical importance of the research. The dissertation work drives a theoretical character. As mentioned above, when it is difficult to investigate the solution of both of the problems for ordinary differential equations and in the problems for special derivative equations, these uninterrupted problems are discretized by the step $h>0$ and brought to the system of algebraic equations. This system is solved. By approaching step h to zero in the solution, it is possible to say a certain opinion about the solution of the uninterrupted problem. Sometimes in the solution of a discrete problem it is possible to obtain an expression for the approximate solution of uninterrupted problem from the expressions obtained at small values of h. It shows the application and practical importance of the work carried. It should be noted that the applications can be applied not only to the above-mentioned problems, but also to finding approximate solutions of problems for integral-differential equations and problems considered for integral equations. It might help to tresearch the solutions of many nonlinear problems.

Approbation and application of the work: 17 scientific works on the theme of the dissertation have been published, 7 of them are scientific articles, 2 are conference materials, and 8 are theses. On the theme of the dissertation the reports have been presented at different scientific conferences, the articles in local and foreign publications and the submitted theses to conferences have been published.

Name of the organization where the dissertation work is presented: The dissertation work has been carried out at the department of Mathematics and Informatics of Lankaran State University.

The total volume of the dissertation with signs, including the volume of the structural chapters of the dissertation separately. The dissertation consists of an introduction, three chapters, a conclusion and a list of used literature. Total volume of the work 192687 signs (title pages - 465 signs, table of contents - 2536 signs, introduction - 61387 signs, chapter I - 240000 signs, chapter II - 30000 signs, chapter III - 72000 signs, conclusion – 2299 signs).

THE CONTENT OF THE WORK

The introduction substantiates the actuality of the dissertation, the object, the subject, the purpose, the tasks and the methods of the research are determined, the scientific innovation, theoretical and practical importance are interpreted, the provisions to be defended are presented.

The first chapter of the dissertation, which is called **“Solution of the problems for differential equations with discrete poverative derivatives till to the third order”,** consists of three paragraphs.

In the first paragraph, which is called, “**The Cauchy and boundary value problems for equation with a discrete poverative derivative up first order"**, after the giving the definitions of discrete additive, discrete multiplicative and discrete poverative integrals, the following problem was considered.

$$y_n^{\{I\}} = f_n,\ n \geq 0, \tag{1}$$

$$y_0 = \alpha. \tag{2}$$

It should be noted that in this area this is the first considered work therefore we started from the problem for the equation with derivative up first order. The general solution of the equation (1):

$$y_n = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0}}}}}},\ n \geq 1, \tag{3}$$

is in form (3). If we take into account the accepted signing for discrete poveartive integral, (3) can be represented in the following form:

$$y_n = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{y_0}}}}}}} \equiv {}^{y_0}\!\left(\int_n^{0} f_k\right) = {}^{y_0}\!\left(\mathrm{P}_{k=n-1}^{0} f_k\right) \tag{4}$$

Then, solution of the Cauchy problem (1), (2):

$$y_n = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\alpha}}}}}}} \equiv \left(\overset{\alpha \quad 0}{\underset{n}{\lrcorner\!\!\int}} f_k \right) = \left(\overset{\alpha \quad 0}{\underset{k=n-1}{\mathrm{P}}} f_k \right) \tag{5}$$

might be in the following form

Now to the equation (1) considering for n-s that meet the condition $0 \le n < m$ let`s solve this equation in the framework of boundary condition.

$$y_0 + \alpha \cdot y_m = \beta, \tag{6}$$

α and β are the given constants here.

For this, if we will write (3) as a general solution of equation (1) in the framework of boundary condition (6), then we obtain:

$$y_0 + \alpha \cdot f_{m-1}^{f_{m-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{y_0}}}}}}} = \beta. \tag{7}$$

From the obtaining equation (7) y_0 should be assigned. By logarithm of this equation, we will obtain:

$$y_0 = \log_{f_0} \log_{f_1} \cdots \log_{f_{m-2}} \log_{f_{m-1}} \frac{\beta - y_0}{\alpha}, \tag{8}$$

taking an arbitrary constant number $\lfloor y_{0_0} \rfloor$ by dint of (8), we set up a subsequence $\{y_{0_k}\}$ in form

$$y_{0_{k+1}} = \log_{f_0} \log_{f_1} \cdots \log_{f_{m-2}} \log_{f_{m-1}} \frac{\beta - y_{0_k}}{\alpha}, \tag{9}$$

It is clear that, according to the assessment obtaining for this subsequence, if this subsequence is monotonic, then it cancels out. Then the limit of this subsequence is taken as y_0.

Thus, we obtain the following theorem:

Theorem 1. For (1) a given equation with a discrete poverative derivative up first order, the solution of the Cauchy problem (1), (2) is given in the (5) form, but solution of the boundary problem (1), (6) can be given by dint of (4), (8).

In the second paragraph of this chapter, which is called **“Research of the solution of problems for equations with a discrete poverative derivative second order"** the following problem was considered.

$$y_n^{\{II\}} = f_n \ , \ n \geq 0, \qquad (10)$$

$$y_0 = \alpha \ , \ y_1 = \beta, \qquad (11)$$

here f_n , $n \geq 0$ is a given subsequence, but y_n , $n \geq 0$ is a searching subsequence, α and β are given numbers.

This solution of the Cauchy problem,

$$y_n = F_{n-1}^{F_{n-2}^{\cdot^{\cdot^{\cdot F_2^{F_1^{F_0}}}}}} \ , \ n \geq 1, \qquad (12)$$

in this form, but F_i -s are set as in (13).

$$F_n = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot f_1^{f_0^{\alpha\sqrt{\beta}}}}}}} \ , \ n \geq 1, \qquad (13)$$

Now to the equation (10), seeing into for $n-$s, meeting to the condition $0 \leq n \leq N-2$

$$y_0^{(I)} = \alpha \ , \ y_n = \beta, \qquad (14)$$

for the solution, meeting to the boundary conditions we obtain this expression.

$$y_n = G_{n-1}^{G_{n-2}^{\cdot^{\cdot^{\cdot G_2^{G_1^{y_1}}}}}} \ , \ n \geq 1, \qquad (15)$$

But for G_i - s this expression exists.

$$G_n = f_{n-1}^{f^{\cdot^{\cdot^{\cdot f_1^{f_0^{\alpha}}}}}} \ , \ n \geq 1, \qquad (16)$$

However for y_1 from the equation

$$\beta = y_n = G_{n-1}^{G_{n-2}^{\cdot^{\cdot^{\cdot G_2^{G_1^{y_1}}}}}} \ , \qquad (17)$$

is obtained this expression. Writing this expression in (15) for the solution of the boundary value problem (10), (14)

$$y_1 = \log_{G_1} \log_{G_2} \cdots \log_{G_{N-1}} \beta\,, \qquad (18)$$

we obtain this expression, since G_i – s are assigned by dint of (16).

$$y_n = \log_{G_n} \log_{G_{n+1}} \cdots \log_{G_{N-1}} \beta, \qquad (19)$$

Thus, we obtain the following theorem:

Theorem 2. If f_n, $n \ge 0$ is a given subsequence, α and β are the constant numbers, then the solution of the Cauchy problem (10), (11) is given by dint of (12) and (13), but the solution of the boundary problem (10),(14) is given through (16), (19).

In this paragraph the solution of both considering problems is obtained in a compact form.

Finally, in the third paragraph **"The Cauchy and boundary value problems for an equation with a discrete poverative derivative third order"**, the following equation was considered.

$$y_n^{\{III\}} = \left(\left(y_n^{\{I\}}\right)^{\{I\}}\right)^{\{I\}} = f_n\,,\ n \ge 0\,, \qquad (20)$$

here f_n , $n \ge 0$ is a given subsequence, but y_n , $n \ge 0$ is a searching subsequence. If we write (20) in an open form, then we obtain the following equation with nonlinear differences:

$$y_{n+3}^{y_{n+2}^{-1+y_{n+1}^{-1}+y_{n+1}^{-1+y_n^{-1}}}} = f_n,\ n \ge 0\,. \qquad (21)$$

for the general solution of this equation

$$y_n = h_{n-1}^{h_{n-2}^{\cdot^{\cdot^{\cdot^{h_3^{h_2^{y_2}}}}}}}\,,\ n \ge 2\,, \qquad (22)$$

the expression was obtained. As,

$$h_n = h_n\left(y_0^{\{II\}}, y_1^{\{I\}}\right) = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{\cdot^{g_2^{g_1^{y_1^{\{I\}}}}}}}}}\,, n \ge 2, \qquad (23)$$

but for g_i -s

$$g_n = g_n\left(y_0^{\{II\}}\right) = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{y_0^{\{II\}}}}}}}}}\,,\ n \ge 1, \qquad (24)$$

the expression is known. So that in (22) – (24) - in the general solution, $y_0^{\{II\}}$, $y_1^{\{I\}}$ and y_2 are arbitrary constants.

If for equation (20),

$$y_k = \alpha_k ,\ k = \overline{0,2} , \qquad (25)$$

the initial conditions are given in this form, then the solution of the Cauchy problem (20), (25) is given through (22) - (24). So that the participating arbitrary constants in them, must be chosen as the following:

$$y_2 = \alpha_2 ,\ y_1^{\{I\}} = \sqrt[\alpha_1]{\alpha_2} ,\ y_0^{\{II\}} = \alpha_2^{\alpha_1^{-1+\alpha_0^{-1}}} . \qquad (26)$$

Now, having looked the equation (20) for n-s, meeting the condition $0 \le n \le N-2$

$$y_0^{\{II\}} = \beta_0 ,\ y_1^{\{I\}} = \beta_1 ,\ y_N = \beta_2 , \qquad (27)$$

let`s solve this equation in the framework of boundary value problems.

For the solution of this equation

$$y_n = \log_{h_n} \log_{h_{n+1}} \cdots \log_{h_{N-2}} \log_{h_{N-1}} \beta_2 , \qquad (28)$$

is gotten such an analytical expression. As,

$$h_n = h_n\left(y_0^{\{II\}}, y_1^{\{I\}}\right) = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{g_2^{g_1^{g_0^{\beta_1}}}}}}} ,\ n \ge 2 , \qquad (29)$$

$$g_n = g_n\left(y_0^{\{II\}}\right) = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{f_2^{f_1^{f_0^{\beta_0}}}}}}} ,\ n \ge 1 ,$$
$$(30)$$

are given through these expressions.

Thus, we obtain the following theorem:

Theorem 3. If $f_n ,\ n \ge 0$ is a given subsequence with the positive element, the α_k- s, if $k = \overline{0;2}$ are positive real numbers, then the solution of the Cauchy problem (20), (25), taking into account (26) in (22), 24), but the solution of the boundary value problem (20), (27) is obtained through (28), (29), (30).

The second chapter **"Problems for equations with mixed discrete derivative second order"** consists of four paragraphs.

In the first paragraph of the second chapter **“The Cauchy and boundary value problems for equation with discrete additivo-poverative derivative”** is researched equation.

$$\left(y_i^{(I)}\right)^{\{I\}} = f_i \ , \ i \geq 0, \tag{31}$$

Here, if f_i, $i \geq 0$, then it is a given subsequence, if y_i, $i \geq 0$ so, becomes to the searching subsequence.

Using the definitions of the discrete poverative derivative, the equation (31) will be in the following form.

$$y_{i+1}^{(I)} = f_i^{\,y_i^{(L)}}, \ i \geq 0, \tag{32}$$

Here setting to i the assessments from 0, will be obtained:

$$y_1^{(I)} = f_0^{\,y_0^{(I)}}, \tag{33}$$

but if $i = 1$, then

$$y_2^{(I)} = f_1^{\,y_1^{(I)}} = f_1^{\,f_0^{\,y_0^{(I)}}}, \tag{34}$$

If we continue this process:

$$y_i^{(I)} = f_{i-1}^{\,f_{i-2}^{\,\cdot^{\cdot^{\cdot^{f_1^{\,f_0^{\,y_0^{(I)}}}}}}}}, \ i \geq 1, \tag{35}$$

Here if we accept the following designation:

$$f_{i-1}^{\,f_{i-2}^{\,\cdot^{\cdot^{\cdot^{f_1^{\,f_0^{\,y_0^{(I)}}}}}}}} \equiv g_i\left(y_0^{(I)}, f_s\right), \ i \geq 1, \tag{36}$$

then, (35) will be in the following form:

$$y_i^{(I)} = g_i\left(y_0^{(I)}, f_s\right), \ i \geq 1, \tag{37}$$

Thus, the composition of equation (31) became one less.

Now, in (37), we use the definitions of the discrete additive derivative:

$$y_{i+1} - y_i = g_i, \ i \geq 1. \tag{38}$$

Here, setting assessments for i, if we add the resulting expressions, then we get:

$$y_i = y_1 + \sum_{k=1}^{i-1} g_k \ , \ i \geq 2, \tag{39}$$

Cauchy problem: For a given equation (31)

$$y_i = \alpha_i, \; i = 0;1, \tag{40}$$

If we add initial conditions, received (31), (40) the solution of the Cauchy problem is formed by dint of (36) and (39).

So that,

$$y_i = \alpha_1 + \sum_{k=1}^{i-1} g_k, i \geq 2. \quad , \; i \geq 1, \tag{41}$$

$$g_i\left(y_0^{(I)}, f_s\right) = g_i\left(y_1 - y_0, f_s\right) = g_i\left(\alpha_1 - \alpha_0, f_s\right) = f_{i-1}^{f_{i-2}^{\cdot^{\cdot^{f_1^{f_0^{\alpha_1-\alpha_0}}}}}}, \; i \geq 1, \tag{42}$$

The boundary value problem. Now in the equation (31), if we accept, $i = \overline{0; n-2}$, and assume, that the boundary conditions are given for this equation.

$$y_0^{(I)} = \beta_0, \; y_n = \beta_1, \tag{43}$$

Then g_i - s by dint of (43) from (36)

$$g_i\left(y_0^{(I)}, f_s\right) \equiv g_i\left(\beta_0, f_s\right) = f_{i-1}^{f_{i-2}^{\cdot^{\cdot^{f_1^{f_0^{\beta_0}}}}}}, \; i \geq 1, \tag{44}$$

turn out to be like this. In order to assign the solutions of the boundary value problem, we must assign y_1 in expression (39).

For this, we take into account (39) in the second boundary conditions (43).

$$\beta_1 = y_N = y_1 + \sum_{k=1}^{n-1} g_k$$

and from here

$$y_1 = \beta_1 - \sum_{k=1}^{n-1} g_k \; .$$

by writing this expression in (39),

$$y_i = \beta_1 - \sum_{k=1}^{n-1} g_k + \sum_{k=1}^{i-1} g_k = \beta_1 - \sum_{k=i}^{n-1} g_k, \tag{45}$$

in the form (31), (43) we obtain solutions of the boundary value problem. Thus, g_i - s have been already assigned in (44).

Thus, we get:

Theorem 4. If f_i, $i \ge 0$ is a given subsequence with the positive limit, but if α_i, $i = 0;1$ are the given real numbers, then there is only one solution of the Cauchy problem (31),(40), and this solution is given through (41), (42), if β_i, $i = 0;1$ are the given positive numbers, then there is only one solution of the boundary value problem (31), (43) and this solution is given through (44), (45).

In the second paragraph **"The Cauchy and boundary value problems for equation with a discrete multiplicativo-poverative derivative"**

$$\left(y_n^{[I]}\right)^{\{I\}} = f_n,\ n \ge 0, \tag{46}$$

for the equation

$$y_0 = \alpha,\ y_1 = \beta, \tag{47}$$

in the framework of the initial conditions for the Cauchy problem and for this equation

$$y_0^{[I]} = \alpha,\ y_m = \beta, \tag{48}$$

in this conditions were considered boundary value problems.

The solution of the Cauchy problem

$$y_n = \beta \cdot \prod_{s=1}^{n-1} g_s,\ \ n \ge 2, \tag{49}$$

$$g_s = f_{s-1}^{f_{s-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\frac{\alpha}{\beta}}}}}}}},\ s \ge 1, \tag{50}$$

by dint of these expressions, but the solution of boundary value problems

$$y_n = \frac{\beta}{\prod_{s=n}^{m-1} g_s},\ n \ge 1, \tag{51}$$

$$g_n = f_{s-1}^{f_{s-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\alpha}}}}}}},\ n \ge 1. \tag{52}$$

we obtain by dint of these expressions.

Thus, we get:

Theorem 5. If $f_n,\ n\geq 0$ is a given subsequence with positive limit, and $\alpha>0,\ \beta>0$ are the given numbers, then (46), (47) the solution of Cauchy problem is obtained via (49) and (50), (46), (48) but the solution of the boundary value problem is obtained through (51) and (52).

In the third paragraph "**Solution of the Cauchy and boundary value problems for equation with a discrete poverativo-additive derivative second order**" for the equation

$$\left(y_n^{\{I\}}\right)^{(I)}=f_n,\ n\geq 0, \tag{53}$$

within the framework of the initial condition was considered

$$y_0=\alpha_0,\ y_1=\alpha_1, \tag{54}$$

The cauchy problem and the same equation (53) if taking into account, that n has $\overline{0;m-2}$ assessments

$$y_0^{\{I\}}=\beta_0,\ y_m=\beta_1, \tag{55}$$

The boundary value problem were considered in these conditions.

The solution of the Cauchy problem:

Via these expressions,

$$g_n\left(y_0^{\{I\}},f_s\right)\equiv g_n\left(\sqrt[y_0]{y_1},f_s\right)=\sqrt[\alpha_0]{\alpha_1}+\sum_{k=0}^{n-1}f_k\ ,\ n\geq 2, \tag{56}$$

and

$$y_n=g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{g_2^{g_1^{\alpha_1}}}}}}\ ,\ n\geq 2, \tag{57}$$

but the solution of the boundary value problems is given via these expressions.

$$g_n\left(y_0^{\{I\}},f_s\right)\equiv g_n(\beta,f_s)=\beta_0+\sum_{k=0}^{n-1}f_k\ ,\ n\geq 1, \tag{58}$$

and

$$y_n=\log_{g_n}\log_{g_{n+1}}\ldots\log_{g_{m-2}}\log_{g_{m-1}}\beta_1,\ n\geq 2, \tag{59}$$

Finally, in the last paragraph of the second chapter **"Problems for equation with a discrete poverativo-multiplicative derivative"** for the equation

$$\left(y_n^{\{I\}}\right)^{[I]} = f_n \ , \ n \geq 0, \tag{60}$$

$$y_0 = \alpha \ , \ y_1 = \beta \ , \tag{61}$$

is considered the Cauchy problem within the framework of the initial condition and the same equation (60), $n = \overline{0; N-2}$ has these assessments

$$y_0^{\{I\}} = \alpha \ , \ y_N = \beta \ , \tag{62}$$

then the solution of the problem has already been researched within the framework of boundary value conditions.

Here (60), (61) the solution of Cauchy problem is given by dint of the following expression

$$y_n = \left(\sqrt[\alpha]{\beta} \cdot f_0 \cdot f_1 \cdots f_{n-2}\right)^{\left(\sqrt[\alpha]{\beta} \cdot f_0 \cdot f_1 \cdots f_{n-3}\right)^{\cdot^{\cdot^{\left(\sqrt[\alpha]{\beta} \cdot \alpha_0\right)^{\beta}}}}} \tag{63}$$

but the solution of the boundary value problem by dint of the expressions (60), (62)

$$\tilde{F}_n \equiv \alpha \cdot \prod_{k=0}^{n-1} f_k \, , \tag{64}$$

and

$$y_n = \log_{\tilde{F}_n} \log_{\tilde{F}_{n+1}} \cdots \log_{\tilde{F}_{N-2}} \log_{\tilde{F}_{N-1}} \beta, \tag{65}$$

The third chapter of the dissertation work is called **“Solution of the problems for differential equations with discrete mixed derivative third order”** consists of six paragraphs. In each paragraph of this chapter are considered the Cauchy and boundary value problems for equations obtained from various combinations of three different discrete derivatives (each equation takes the third derivative). We note again that the open notation of each of these equations reduces to very complex nonlinear equations. We will provide analytical solutions for these problems.

In the first paragraph of the third chapter, entitled **“The Cauchy and boundary value problems for differential equation with a discrete additivo-multiplicativo-poverative derivative”** for equation

$$\left(\left(y_n^{(I)}\right)^{[I]}\right)^{\{I\}} = f_n,\ n \geq 0, \tag{66}$$

$$y_0 = \alpha_0,\ y_1 = \alpha_1,\ y_2 = \alpha_2, \tag{67}$$

the Cauchy problem is considered within the framework of the initial conditions. Here if $f_n,\ n \geq 0$ is given sequence, but if y_n, $n \geq 3$ then is a searching subsequence, are known α_0, α_1 and α_2 and given y_0, y_1 and y_2.

This (66), (67) solution of the Cauchy problem

$$g_n = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f^{\frac{\alpha_2-\alpha_1}{\alpha_1-\alpha_0}}}}}}}}, \tag{68}$$

$$h_n = (\alpha_1 - \alpha_0)\cdot\prod_{k=0}^{n-1} g_k, \tag{69}$$

and

$$y_n = \alpha_0 + \int_0^n h_k,\ n \geq 3, \tag{70}$$

is given using expressions.

Now, the considering equations (66), if $n = \overline{0; m-3}$ has got these assessments, then for this equation

$$\left(y_0^{(I)}\right)^{[I]} = \alpha,\ y_1^{(I)} = \beta,\ y_m = \gamma, \tag{71}$$

let`s take, that the boundary conditions are given. Then the solution of this boundary value problem

$$G_n = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\alpha}}}}}}},\ n \geq 1, \tag{72}$$

$$H_n = \beta\cdot\prod_{s=1}^{n-1} G_s,\ n \geq 2, \tag{73}$$

and

$$y_n = \gamma - \sum_{k=n}^{m-1} H_k \quad , n \geq 3, \tag{74}$$

is given by dint of these expressions.

So we get:

Theorem 6. If f_n, $n \geq 0$ is a subsequence with positive limit, if α_0, α_1 and α_2 being positive numbers, and $\alpha_0 \neq \alpha_1$, so, then (66), (67) is the only solution of the Cauchy problem and this solution is given through expressions (68)-(70), if f_n together with the sequence, if α, β and γ are given positive numbers, then (66), (71) is the only solution of the boundary value problem and this solution is given through expressions (72)-(74).

In the second paragraph of the third chapter, which is called **"The Cauchy and boundary value problems for differetial equation with a discrete additivo-poverativo-multiplicative derivative"**

$$\left(\left(y_n^{(I)}\right)^{\{I\}}\right)^{[I]} = f_n, \ n \geq 0, \tag{75}$$

for the equation

$$y_0 = \alpha, \ y_1 = \beta, \ y_2 = \gamma, \tag{76}$$

the Cauchy problem was considered within the framework of the initial conditions. This is the solution of the Cauchy problem

$$y_n = \gamma + \sum_{k=2}^{n-1} h_k, \ n \geq 3, \tag{77}$$

$$h_n = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{g_2^{g_1^{\gamma-\beta}}}}}} \quad , n \geq 2, \tag{78}$$

and

$$g_n = \sqrt[\beta-\alpha]{\gamma - \beta} \cdot \prod_{s=0}^{n-1} f_s, \ n \geq 2, \tag{79}$$

is given by dint of these expressions.

Now looking at assessments of the equation (75) considering in $n = \overline{0; m-3}$ for this equation let`s take, that boundary conditions are shown as follows:

$$y_1^{(I)} = \alpha \quad \left(y_0^{(I)}\right)^{\{I\}} = \beta\,,\ y_m = \gamma\,. \tag{80}$$

Then this is the solution of the boundary value problem:

$$y_n = \gamma - \sum_{k=n-1}^{m-2} F_k^{F_{k-1}^{F_{k-2}^{\cdot^{\cdot^{\cdot F_2^{F_1^{\alpha}}}}}}}\,,\ n \ge 2, \tag{81}$$

and

$$F_n = \beta \cdot \prod_{k=0}^{n-1} f_k\ ,\ n \ge 1, \tag{82}$$

is given via these expressions.

Thus, we obtain the following theorem:

Theorem 7. If in the equation (75) is f_n, $n \ge 0$, then the subsequence with positive limit, (76) is given in the initial condition α, β, γ if are positive, if $\beta > \alpha$, $\gamma > \beta$, then there is the only solution of the Cauchy problem and this solution is given through (77) and (79), if we consider the equation (75) in assesments $n = \overline{0; m-3}$, f_n-s being positive, if in (80) datas α, β, γ - if are positive, then (75), (80) are the only solution of the boundary value problem and this solution is given through expressions (81) and (82).

In the third paragraph of the third chapter, entitled **"The Cauchy and boundary value problems for differential equation with discrete multiplicativo-additivo-poverative derivative"**

$$\left(\left(y_n^{[I]}\right)^{(I)}\right)^{\{I\}} = f_n,\ n \ge 0, \tag{83}$$

for the equation

$$y_k = \alpha_k\,,\ \ k = \overline{0{,}2}\,, \tag{84}$$

The Cauchy problem was considered within the framework of the initial condition. This is the solution of the Cauchy problem

$$y_n = \alpha_2 \cdot \prod_{s=2}^{n-1} h_s\left(\frac{\alpha_2}{\alpha_1} - \frac{\alpha_1}{\alpha_0}, \frac{\alpha_2}{\alpha_1}\right),\ n \ge 3, \tag{85}$$

$$h_n\left(y_1^{[I]}, \left(y_1^{[I]}\right)^{(I)}\right) \equiv h_n\left(\frac{\alpha_2}{\alpha_1} - \frac{\alpha_1}{\alpha_0}, \frac{\alpha_2}{\alpha_1}\right) = \frac{\alpha_2}{\alpha_1} \cdot \sum_{s=1}^{n-1} g_s\left(\frac{\alpha_2}{\alpha_1} - \frac{\alpha_1}{\alpha_0}\right),\ n \ge 2, \tag{86}$$

and

$$g_s\left(y_0^{[I]}\right)^{(I)} \equiv g_n\left(\frac{\alpha_2}{\alpha_1}-\frac{\alpha_1}{\alpha_0}\right)= f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\frac{\alpha_2}{\alpha_1}-\frac{\alpha_1}{\alpha_0}}}}}}}} \quad , \ n \geq 1, \tag{87}$$

is given through this expression.

By the same rule to the equation (83), for assessments

$$\left(y_0^{[I]}\right)^{(I)} = \beta_0 , \ y_1^{[I]} = \beta_1 , \ y_N = \beta_2 , \tag{88}$$

considering the problem in the framework of boundary conditions, the solution of this boundary value problem

$$h_n\left(\beta_0, \beta_1\right) = \beta_1 + \sum_{s=1}^{n-1} g_s\left(\beta_0\right), \ n \geq 2, \tag{89}$$

$$g_n\left(\beta_0\right) = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\beta_0}}}}}}} \quad , \tag{90}$$

and

$$y_n = \frac{\beta_2}{\prod_{s=n}^{N-1} h_s\left(\beta_0, \beta_1\right)} \ , \ n \geq 3 . \tag{91}$$

is given through expressions.

Thus, we obtain the following theorem:

Theorem 8. In the equation (83), if f_n , $n \geq 0$, then the subsequence with positive limit, if in (84) the given α_k-s are positive, then the solution of the Cauchy problem is given through the expressions (85)-(87), the same for the equation (83) for n=$\overline{0, N-3}$, if β_0, β_1, β_2-s are positive, where they are given in (88), then within the framework of the boundary conditions, the solution of the problem is given through the expressions (89)-(91).

In the fourth paragraph of the third chapter **"Solution of the Cauchy and boundary value problems for differential equation with discrete multiplicativo-poverativo-additive derivative"**

$$\left(\left(y_n^{[I]}\right)^{\{I\}}\right)^{(I)} = f_n,\ n \geq 0, \qquad (92)$$

for the equation

$$y_0 = \alpha_0,\ y_1 = \alpha_1,\ y_2 = \alpha_2, \qquad (93)$$

the Cauchy problem was considered within the framework of initial conditions. An analytical solution of this problem

$$g_n\left(\left(y_0^{[I]}\right)^{\{I\}}\right) = \sqrt[\frac{\alpha_1}{\alpha_0}]{\frac{\alpha_2}{\alpha_1}} + \sum_{k=0}^{n-1} f_k\ ,\ n \geq 1, \qquad (94)$$

$$h_n\left(y_1^{[I]}\right) = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{\cdot g_1^{\frac{\alpha_1}{\alpha_0}}}}}}\ ,\ n \geq 2, \qquad (95)$$

and

$$y_n = \alpha_1 \cdot \prod_{s=1}^{n-1} h_s\ ,\ n \geq 3, \qquad (96)$$

is given through expressions. Now to the equation (92) in assessments $n = \overline{0; N-3}$

$$\left(y_0^{[I]}\right)^{\{I\}} = \beta_0,\ y_1^{[I]} = \beta_1,\ y_N = \beta_2, \qquad (97)$$

if will be considered within the framework of boundary conditions, then this is the solution of boundary value problem

$$g_n\left(\left(y_0^{[I]}\right)^{\{I\}}\right) = \beta_0 + \sum_{k=0}^{n-1} f_k, \qquad (98)$$

$$h_n\left(y_1^{[I]}\right) = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{\cdot g_1^{\beta_1}}}}}\ , \qquad (99)$$

and

$$y_n = \frac{\beta_2}{\prod_{s=n}^{N-1} h_s}, \qquad (100)$$

is given through these expressions. And in the same way, we get the following theorem:

Theorem 9. If in the equation (92), f_n, $n \geq 0$ is so, then the given subsequence with positive limit, in the initial condition (93), if α_0, α_1, α_2 are positive numbers, then (92), (93) are the solution of the Cauchy problem is given through expressions (94)-(96), an equation (92),which is considered in assessments for $n=\overline{0;N-3}$, by positive β_0, β_1, β_2 is given through expressions (98)-(100) within the boundary conditions (97).

In the fifth paragraph of the third chapter, which is called **"Research of the Cauchy and boundary value problems for differential equation with discrete poverativo-additivo-multiplicative derivative"**

$$\left(\left(y_n^{\{I\}}\right)^{(I)}\right)^{[I]} = f_n,\ n \ge 0, \qquad (101)$$

for the equation

$$y_k = \alpha_k,\ k = \overline{0;2}, \qquad (102)$$

the Cauchy problem was considered within the framework of the initial conditions and the solution of this problem in an analytical form

$$g_n(\alpha_0, \alpha_1, \alpha_2) = \left(\sqrt[\alpha_1]{\alpha_2} - \sqrt[\alpha_9]{\alpha_1}\right) \cdot \prod_{k=0}^{n-1} f_k,\ n \ge 1, \qquad (103)$$

$$h_n(\alpha_1, \alpha_2) = \sqrt[\alpha_1]{\alpha_2} + \sum_{s=0}^{n-1} g_s,\ n \ge 2, \qquad (104)$$

and

$$y_n = h_{n-1}^{h_{n-2}^{\cdot^{\cdot^{\cdot h_3^{h_2^{\alpha 2}}}}}},\ n \ge 3, \qquad (105)$$

is given through expressions. By the same rule, if to the equation (101) for the assessments $n=\overline{0;N-3}$

$$y_1^{\{I\}} = \beta_0,\ \left(y_0^{\{I\}}\right)^{(I)} = \beta_1,\ y_N = \beta_2, \qquad (106)$$

If, it will be considered within the framework of boundary conditions, the analytical expression of this problem

$$g_n(y_0, y_1, y_2) = \beta_1 \cdot \prod_{k=0}^{n-1} f_k, \qquad (107)$$

$$h_n(y_1, y_2) = \beta_0 + \sum_{s=1}^{n-1} g_s, \qquad (108)$$

and

$$y_n = \log_{h_n} \log_{h_{n+1}} \cdots \log_{h_{N-1}} \beta_2, \qquad (109)$$

is given through expressions.

Thus, we get:

Theorem 10. If in the equation (101) the given f_n, $n \geq 0$, then the positive subsequence, α_0, α_1, α_2 are being positive (101), (102) are the solution of the Cauchy problem through (103)-(105), the equation (101), considering for assessments $n = \overline{0; N-3}$, β_0, β_1, β_2 are being positive, the solution (106) within the boundary conditions is given through expressions (107)-(109).

Finally, in the last paragraph of the third chapter **"The Cauchy and boundary value problems for differential equation with discrete poverativo-multiplicativo-additive derivative"** for the equation

$$\left(\left(y_n^{\{I\}}\right)^{[I]}\right)^{(I)} = f_n,\ n \geq 0, \qquad (110)$$

$$y_k = \alpha_k,\ k = \overline{0;2}, \qquad (111)$$

the Cauchy problem was considered in the initial conditions. This is the solution of the problem

$$g_n = \frac{\sqrt[\alpha_1]{\alpha_2}}{\sqrt[\alpha_0]{\alpha_1}} + \int_0^n f_k \ , \quad n \geq 0, \qquad (112)$$

$$h_n = \sqrt[\alpha_1]{\alpha_2} \cdot \int_0^n g_m \ , \ n \geq 1, \qquad (113)$$

and

$$y_n = \int_n^0 h_k \ , n \geq 3, \qquad (114)$$

is given through these expressions.

If to the equation (110) for $n = \overline{0; N-3}$ assessments

$$y_1^{\{I\}} = \alpha \ , \ \left(y_0^{\{I\}}\right)^{[I]} = \beta \ , \ y_N = \gamma \ , \qquad (115)$$

If will be considered within the framework of boundary conditions, this is an analytical expression for the solution of the boundary value problem

$$g_n = \beta + \int_0^n f_k \ , \qquad (116)$$

$$h_n = \alpha \cdot \int_0^n g_m \ , \qquad (117)$$

and

$$y_n = \log_{h_n} \log_{h_{n+1}} \cdots \log_{h_{N-2}} \log_{h_{N-1}} \gamma \ , \qquad (118)$$

is given through these expressions.

Thus, we get:

Theorem 11. If f_n , $n \geq 0$ the given subsequence with the positive limit, if the given datas of initial and boundary conditions are positive numbers, then the solution of the Cauchy problem is given through (112)-(114), and the solution of the boundary value problem is given through these expressions (116)-(118).

CONCLUSION

The following are the results and generalizations obtained from the research in the dissertation.

In the dissertation “Discrete additive, multiplicative and poverative derivative concepts and the applications to solution of the Cauchy and boundary value problems for differential equations with discrete derivative”, which consists of introduction, three chapters, conclusion and literature, research of solution of the Cauchy and boundary value problems for equations with discrete derivative up third order was discussed.

The introduction begins from the information about additive, multiplicative and poverative derivatives and integrals in case of continuity. The additive integral has been known since ancient times, the additive derivative appeared in the time of I. Newton and G. Leibniz. Although multiplicative derivatives and integrals were created 70-80 years ago, problems for such equations have only recently begun to be considered. But poverative derivatives and integrals are being considered by us. Since seven algebraic operations were not enough for this, a new direct and inverse operation was assigned. Thus, in the case of continuity, each derivative is given by two consecutive inverse operations, and each integral is given by two consecutive direct operations in a discrete way, the derivative is given by only one inverse operation and the discrete integral by one direct operation. Therefore, no new operation is required to give a discrete derivative and an integral. That is, the seven algebraic operations we know are sufficient to give a discrete derivative and an integral. In the first chapter of the dissertation, only the poverative derivatives, the Cauchy and boundary value problems were considered for the equations (first, second and third order) and were obtained the analytical expressions for their solution.

In the second chapter, the Cauchy and boundary value problems were considered for the equations with different discrete derivative second order and were obtained analytical expressions for their solution.

Finally, the third chapter the Cauchy and boundary value problems were considered for three different discrete derivative equations and were obtained analytical expressions for their solution.

It should be noted that in this direction the considering work is a result obtaining from previously.

The method of research is the same. Thus, the general solution of the previously considering work (the solution of which depends on an arbitrary constant until order) is assigned. These equations are equations with very complex nonlinear differences. The arbitrary constants included in the general solution are assigned from the initial or boundary conditions. Thus, the analytical expressions are obtained for considering solution of the problems.

It should be noted that in the work not only the results, also the problems are new. As we have said above, when we write these problems openly, it becomes clear how complex the differences are in terms of nonlinear conditions for equations.

Although the work is theoretical, it can also be used for approximate solutions.

The main results of the dissertation were published in the following scientific works:

1. Diskret yeni törəmənin xassələri // "Müasirləşən Azərbaycan: Yeni yüksəliş mərhələsi" mövzusunda keçirilən gənc tədqiqatçıların Respublika Elmi Konfransının Materialları, – Lənkəran, – 2017. – s. 29-30.
2. İkinci tərtib diskret poverativ törəməli tənlik üçün məsələlərin həlli // – Lənkəran: Lənkəran Dövlət Universitetinin Elmi Xəbərləri, Təbiət elmləri – 2018. №1, – s. 55-58.
3. Solution of Cauchy and boundary problems for the third compilation discrete additive-multiplicative-powerative derivative equation // – Ukraina, Vestnik Київського Національного Університету Імені Тараса Шевченка, Серія Фізико-Математичні Науки, – 2018. №1, pp. 50-54.

4. Diskret poverativo-multiplikativ törəməli tənlik üçün məsələlər // – Bakı, Azərbaycan Texniki Universiteti, Texnika Elmləri, Elmi əsərlər, – 2018. №2. – s. 90-94.

5. On a solution of the Cauchy problem for the discrete equation with powerative-multiplicative-additive derivatives // XXXI İnternational Conference Problems of Decision Making Under Uncertainties (PDMU-2018) Abstracts, – Republic of Azerbaijan, – Lankaran, – 03-07 July, – 2018. – pp 16-17.

6. Solution of Cauchy problem for third discrete derivative additive-multiplicativo-poverativo derivative equation // XXXII International Conference Problems of Decision Making Under Uncertainties (PDMU-2018), – Czech Republic, – Pragua, –24 august– 01 september, – 2018. – pp.84-86.

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8. İkinci tərtib diskret multiplikativo-poverativ törəməli tənlik üçün Koşi və sərhəd məsələsinin həlli // "İnteqrasiya mühitində Azərbaycan elminin qarşısında duran vəzifələr" mövzusunda Respublika Elmi Konfransının materialları, – Lənkəran, Lənkəran Dövlət Universiteti, – 23-24 dekabr, – 2018. – s. 24-25.

9. Yeni birinci tərtib diskret poverativ törəməli tənlik üçün Koşi və sərhəd məsələlərinin həlli // – Lənkəran Dövlət Universiteti, Elmi Xəbərlər, Təbiət bölməsi, – 2018. №2. s. 46-50.

10. Birinci tərtib diskret poverativ törəməli tənlik üçün Koşi və sərhəd məsələlərinin həlli // Professor Nihan Əliyevin 80 illik yubileyinə həsr olunmuş "Riyaziyyat elminin inkişafının yeni mərhələsi" mövzusunda Universitet elmi Konfransının Materialları, – Lənkəran, – 2019. – s. 61-63.

11. İkinci tərtib diskret poverativo-additiv törəməli tənlik üçün Koşi məsələsinin həlli // Tədris prosesində elmi innovasiyalarin tətbiqi yolları mövzusunda Ümummilli Lider Heydər Əliyevin anadan olmasının 96-cı ildönümünə həsr olunmuş Respublika elmi-praktik konfransı, – Lənkəran, – 7-8 may, – 2019. – s. 59-60.

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17. III tərtib diskret additivo-poverativo-multiplikativ törəməli tənlik üçün Koşi və sərhəd məsələlərinin həllinin araşdırılması // – Bakı: Bakı Universitetinin Xəbərləri, fizika-riyaziyyat elmləri seriyası – 2021. №2, – s. 53-57.

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